Master IDIL M1 internship project

Exploration of qualitative trajectories for robust validation of ecosystem modeling and development of a numerical tool

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In short: Develop a methodology to determine the set of possible **qualitative** trajectories of ecological models, as a tool for robust model validation. Application to generic "resource-consumer" models with one or several species (systems of deterministic differential equations).
The subject is part of an ongoing research with J. Harmand & E. Le Quémener (LBE) and C. Gaucherel (AMAP).

Context

When modeling population dynamics, a usual approach is to propose an analytical model (with some parameters) and then to look for its validation on *quantitative* data (nonlinear regression, accept-reject test...)
In face of poor or not very accurate data, we propose to look for a qualitative validation, based on qualitative observations of the transients, such as time successions of growths or declines of variables (very few tools have been developed in the literature compared to quantitative methods).

Objectives

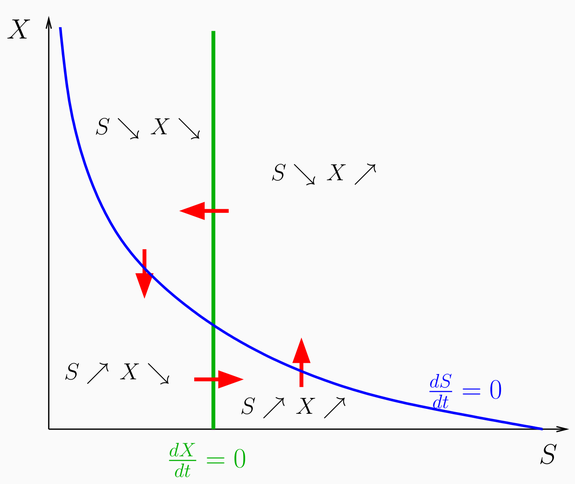
- Propose relevant definitions of qualitative states (positive or negative derivatives, relative position in the phase portrait...) for discriminating between models.
- Study the transitions graph between qualitative states for a given model.
- Analyze (theoretical and numerically) all the possible temporal successions of transitions.
- Test on synthetic data the robustness of discrimination between models.

An example: the chemostat model (see ref. [1])

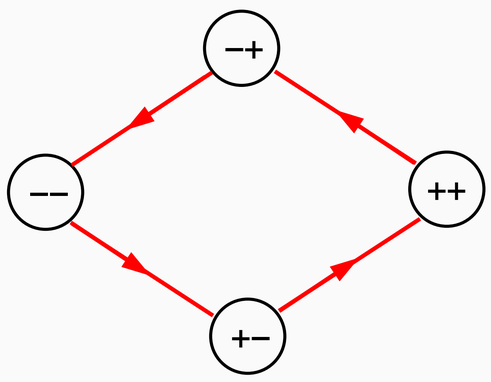
$$\textit{substrat:}\quad \frac{dS}{dt} = -\frac{1}{Y}\frac{SX}{K+S} + D(S_{in} - S)$$

$$\textit{bacteria:}\quad \frac{dX}{dt} = \frac{SX}{K+S} - DX$$

▶ 1. Phase portrait



▶ 2. Transitions graph



▶ 3. Qualitative trajectories

only six cases are possible:

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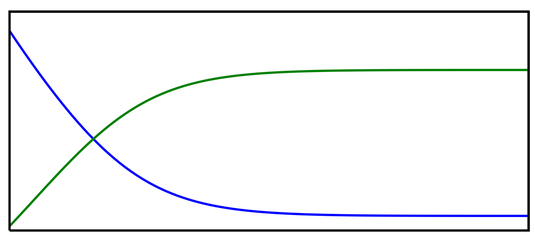

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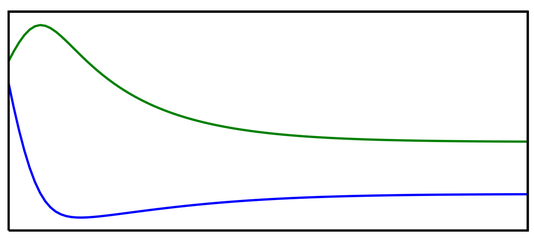

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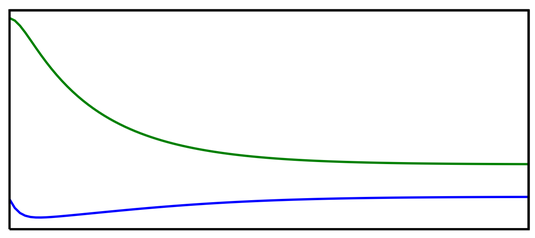

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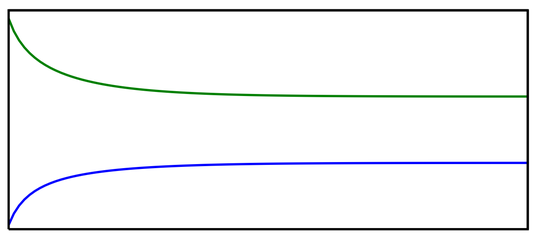

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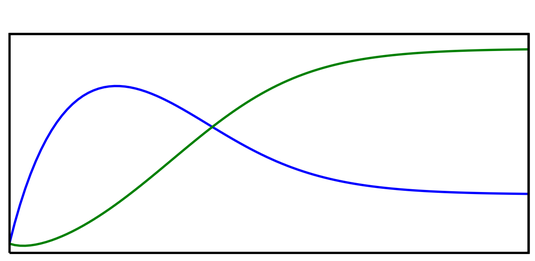

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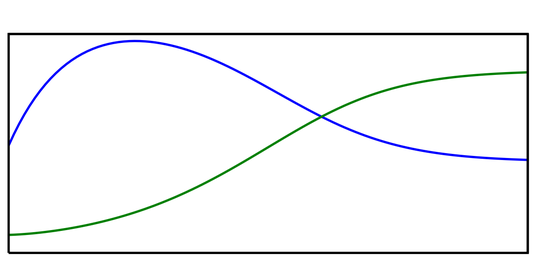

Prerequisites: theory of ordinary differential equations, numerical integration with Matlab, Scilab, Pyhton, Julia...

References:

[1] Harmand, Lobry, Rapaport, Sari, *The Chemostat: Mathematical Theory of Microorganisms Cultures*, Wiley, 2017.
[2] Bernard, Gouzé, *Global qualitative description of a class of nonlinear dynamical systems*, Artificial Intelligence 2002, 136(1).