Modeling growth and death processes of populations

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In biology, one is frequently confronted with the random evolution in time of a small number of entities. This is obviously not described by naive differential equations such as the logistic equation since quantities of interest are discrete. Stochastic calculus aims to correctly describe such situations using simulations or computing the evolution of different statistical quantities such as the mean, variance, and higher-order correlations analytically.

In this project, we will build progressively from simple to more complex situations, describing the birth and death of individuals, growth, interactions, and extinctions of populations, using a variety of methods (master equation, Langevin equation, mean field approximation, Fokker-Planck equation, and different techniques of numerical simulations).

Although these methods are classics (and form a strong background for an apprentice model builder), some aspects are left rather undocumented except by specialists, such as the correct behavior of a self-interacting population near extinction.

The final question we want to address is to compare the behavior of a master equation having a logistic equation as a mean-field approximation with the logistic equation with noise (frequently used in theoretical biology). Of particular interest is the distribution of extinction times which can be related to individual temporal trajectories of the populations. If time permits, the problem will also be addressed on a spatial lattice.

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