Proposal of internship project
– Exploration of qualitative trajectories for robust validation of ecosystem modeling and development of a numerical tool –

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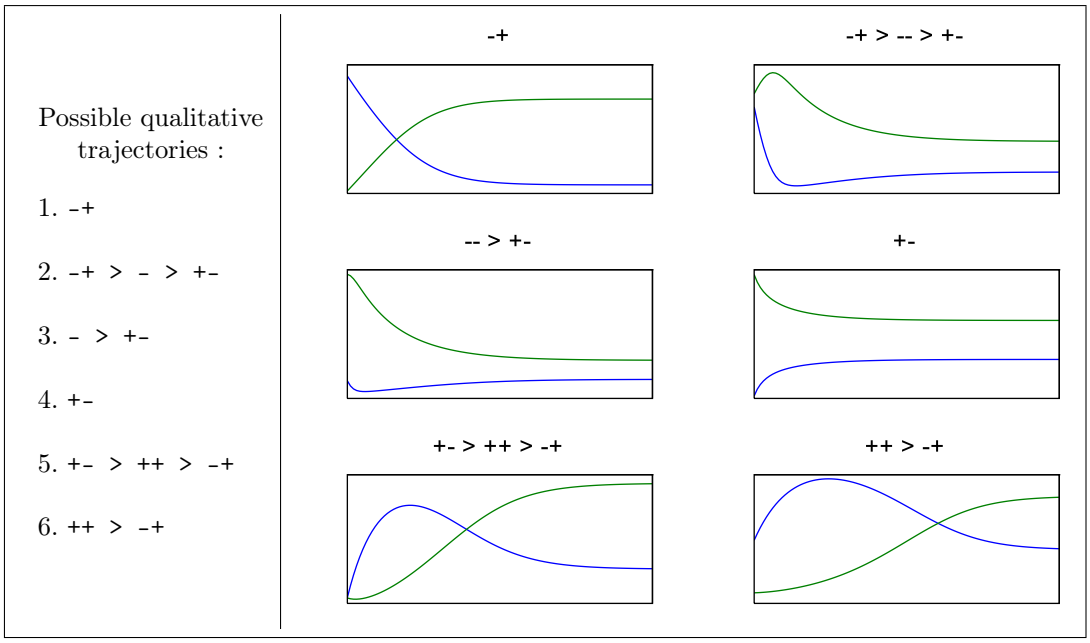
Short Description : Develop a methodology to determine the set of possible qualitative trajectories of a given class of (deterministic) ecological models, as a tool for robust model validation. The considered models will be dynamical systems based on ordinary differential equations.

When modeling population dynamics or more generally ecosystems, a usual approach is to propose an analytical model (typically a system of ordinary differential equations with some parameters) and then to look for a validation of this model on available experimental data. Typically, nonlinear regression is used to look for the best set of parameters of the model and then depending on the quality of data adjustment, a decision has to be taken : accept or reject the model. The decision can be also taken among a set of different models or classes of models to be chosen. Sometimes, data are too poor or not accurate enough to obtain a strong validation. The analysis of some theoretical properties of the models, such as their asymptotic behaviors or the monotony of some variables in the transients could bring additional information that could strengthen the validation. But this is far to be a systematic approach. On the other hand, discussions with biologists or ecologists can report qualitative observations on the real-life dynamics, such as for instance time successions of growths or declines of variables that practitioners could consider to be robust. These qualitative observations are some how hidden on the data but can be easily extracted. However, they are rarely exploited because to our knowledge there are very few qualitative approaches compared to quantitative ones (excepted the few references cited below [2, 3, 4, 5]). The study of compatibility of qualitative observations with possible qualitative behaviors of the model can be then a robust first qualitative validation (or invalidation) of the model, prior to any parameters calibration.

Let us illustrate what could be qualitative observations on the "chemostat" resource-consumer model (see e.g. [1]) :

$$\frac{dS}{dt} = -\frac{1}{Y}\mu(S)X + D(S_{in} - S),$$
$$\frac{dX}{dt} = \mu(S)X - DX,$$

where S and X stand for the resource and consumer densities, D the removal rate, S_{in} the concentration of resource feed, Y the yield conversion of the resource and $\mu(\cdot)$ the specific growth rate. If one considers that qualitative states out of equilibria $++$, $+-$, $--$, $-+$, where the sign $\pm$ denotes a variable (S or X) that is time increasing/decreasing (the first sign for the variable S and the second for X), it can be shown that for an increasing function $\mu(\cdot)$, the only sequences of qualitative states (that we shall call "qualitative" trajectories) from any positive initial condition is one among the following six cases, as illustrated below. If the experimental data do not fall in one of these 6 cases, then we invalidate the model.



The objective of the internship is to propose a systematic way to obtain qualitative trajectories from a given (parameterized) model. The qualitative state can be simply increasing/decreasing property but other qualitative properties could be incorporated into the state definition, such as position in orthants relatively to the steady state [4, 5]... The search for all possible transitions among states and qualitative trajectories can be determined with pure numerical simulations, but it can be also guided with some analytical determinations, such as the geometry of the isoclines. Some thoughts are also awaited to detect oscillations and limit cycles, and to propose a convenient way to represent qualitatively these situations. The proposed method will be tested on synthetic data generated by the models and corrupted with some noise, to test the robustness of in/validation method.

For the duration of the internship, the focus will be on two-dimensional Kolmogorov systems with perspectives for high dimensions. The prerequisite knowledge is the classical theory of systems of ordinary differential equations, and good skills in numerical simulations with software such as Matlab, Scilab, Pyhton, Julia... Depending on the background and the motivations of the candidate, the internship can take a more theoretical or more applied direction. The subject if part of an ongoing research work conducted by A. Rapaport (MISTEA), J. Harmand, E. Le Quémener (LBE) and C. Gaucherel (AMAP) with whom the candidate can interact.

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Références

[1] Harmand J., Lobry C., Rapaport A., Sari T. *The Chemostat : Mathematical Theory of Microorganisms Cultures*, Wiley, 2017.

[2] Dordan, O. *Mathematical problems arising in qualitative simulation of a differential equation*, Artificial Intelligence 1992, 55, 61-86.

[3] Bernard, O., Gouzé, J.-L. *Transient Behavior of Biological Loop Models with Application to the Droop Model*, Math. Biosci. 1995, 127(1), 19-43.

[4] Bernard, O., Gouzé, J.-L. *Transient behavior of biological models as a tool of qualitative validation - Application to the droop model and to a N-P-Z model*, Journal of Biological Systems 1996, 4(3) 303-314.

[5] Bernard, O., Gouzé, J.-L. *Global qualitative description of a class of nonlinear dynamical systems*, Artificial Intelligence 2002, 136(1), 29-59.