Modelling Growth and Death of Populations.

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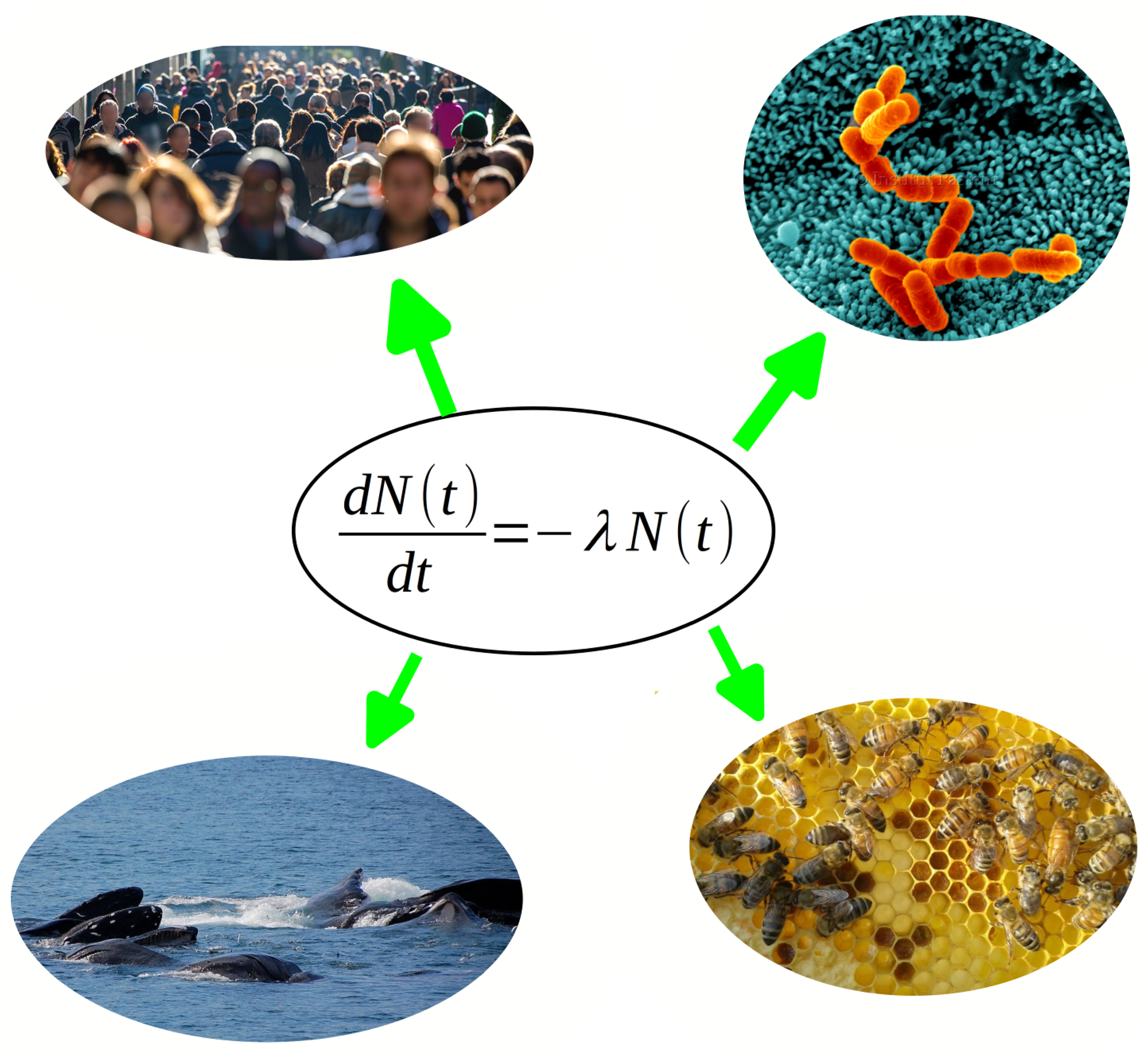
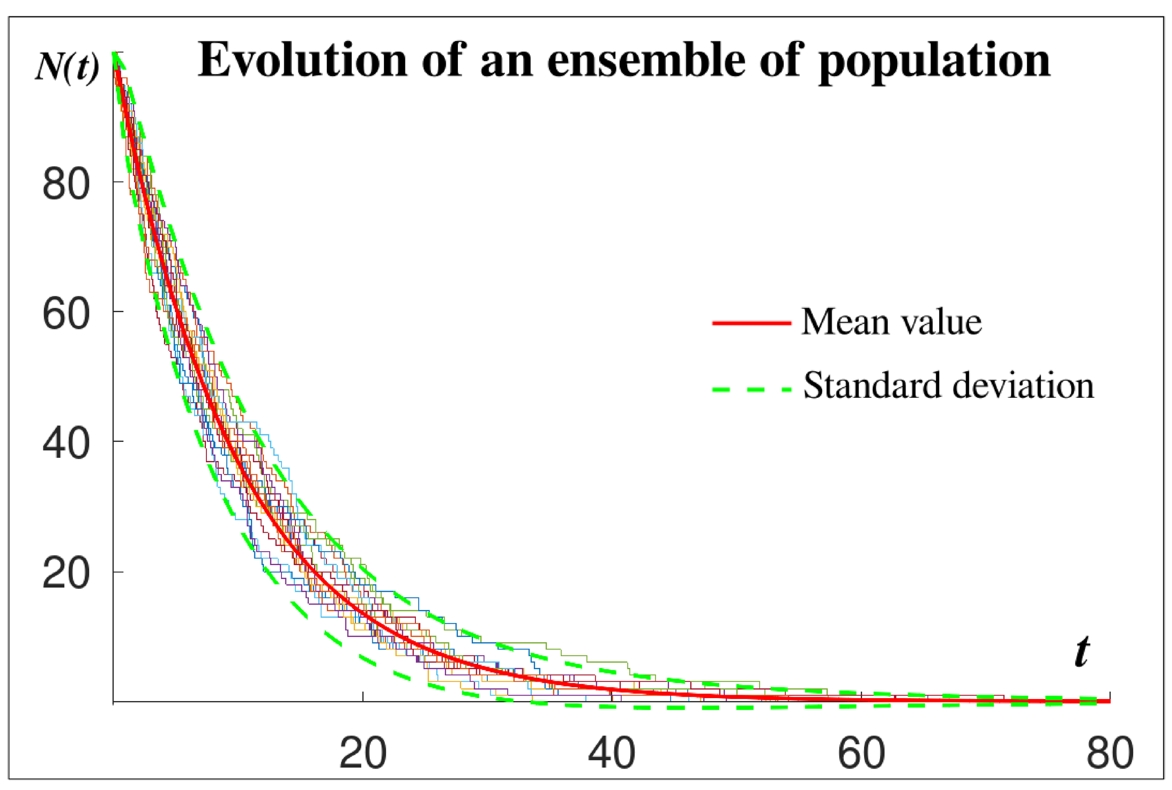
1. Introduction

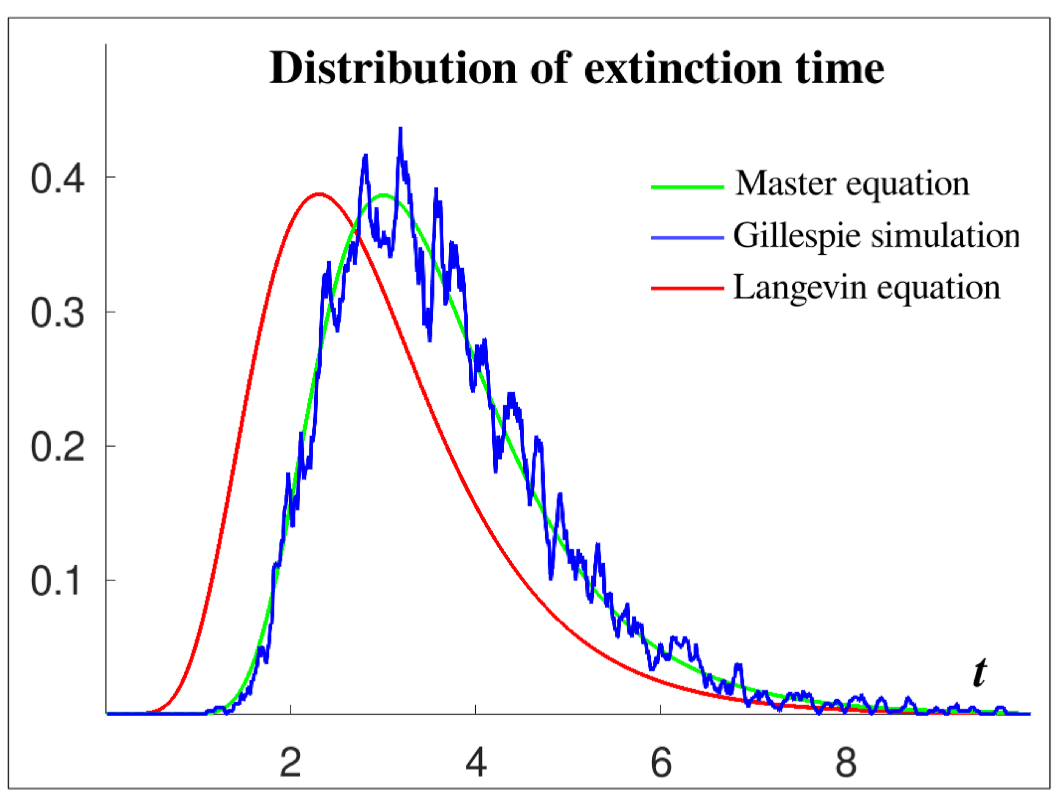
In ecology one is frequently confronted to small number of entities belonging to coupled populations evolving in space and time. These problems are obviously not described by naive differential equations such as the logistic equation, since quantities of interest are discrete and evolve in a non deterministic way. **Stochastic calculus** aims at describing such situations, either by using **simulations or analytical methods** to compute the evolution of different population-related statistical quantities such as the mean and variance of the number of individuals, or the probability of extinction [1] [2] [3].



2. An example

As an example, we show below (left) the time-histories of an ensemble of populations corresponding to the simple equation given in the box "Introduction". Each population evolves **randomly**, but these different histories produce an ensemble with **a definite mean value and standard deviation at each time**.
Various interesting quantities can be obtained. For example, each population will get extinct at a different time, and one can obtain **the distribution of extinction times**. This is shown below (right) for populations each containing initially 10 individuals (green : theory, blue: simulation, red: approximation of the Langevin equation).





5. Perspectives

The above example gives a flavour of what the project is about: the equations can be made more complex and more realistic adlib, to include **interacting and competing species** [4], **diffusion of the population in space, finite resources** [5], **aging** ... It therefore contains all the tools used by theoreticians and model designers in fields as varied as ecology, cell biology, demography, epidemiology, waiting queues and traffic models.
Supervisors interests range **from pure theory to applied physical biology**. Topics can be adapted to student tastes, be them numerical, theoretical, applied or fundamental.

3. Stochastic methods

Different methods are used to describe quantitatively evolving populations with various degrees of complexity and accuracy, such as :

- Master equation
- Mean field approximation
- Gillespie algorithm
- Langevin equation
- Fokker-Planck equation
- ...

4. Goal of the Project

In this project, we will build progressively from simple to more complex situations, describing **birth and death of individuals, growth, interactions and extinctions of populations**, using a variety of methods cited in box (3).

Although these methods are classics (and form a strong background for an apprentice model builder) some aspects are left rather undocumented/ unknown, such as the correct behavior of a self interacting population near extinction, see example in box (2).

6. References

[1] T.J. Newman, Jean-Baptiste Ferdy, and C. Quince. Extinction times and moment closure in the stochastic logistic process. *Theoretical Population Biology*, 65(2):115–126, 2004.

[2] Bruno Anselme. *Biomathématiques*. Dunod Sciences Sup, Paris, 2015.

[3] O. Ovaskainen and B. Meerson. Stochastic models of population extinction. *Trends in Ecology & Evolution*, 25(11):643–652, 2010.

[4] Tal Agranov and Guy Bunin. Extinctions of coupled populations, and rare event dynamics under non-gaussian noise. *Phys. Rev. E*, 104:024106, Aug 2021.

[5] L.R. De Oliveira, C. Castellani, and G. Turchetti. One parameter family of master equations for logistic growth and bcm theory. *Communications in Nonlinear Science and Numerical Simulation*, 20(2):461–468, 2015.