IDIL internship proposal
– Contribution of automatic control to epidemiology –

MISTEA research unit, Campus La Gaillarde, Montpellier, France

2024-25 academic year

Short Description. Study the synthesis of state estimators and control laws for decision making in epidemiology. The considered models will be dynamical systems based on ordinary differential equations, such as SIR, SIS, SIRS... and other models.

Automatic control consists in considering "input-output" models of the form

$$\frac{dx(t)}{dt} = f(x(t), u(t)), \quad y(t) = h(x(t)), \quad x(t) \in \mathbb{R}^n, \ u(t) \in U \subset \mathbb{R}^p, \ y(t) \in \mathbb{R}^q$$

where $u(\cdot)$ and $y(\cdot)$ are the "input" and "output" vectors. The two main main goals are

1. design controls law $t \mapsto u(t)$ or feedback controls $x \mapsto \phi(x)$ so that the closed-loop system

$$\frac{dx(t)}{dt} = g(x(t))) = f(x(t), \phi(x(t))$$

   possesses *good* properties, such as global asymptotic stability of all the solutions or optimization of a criterion

$$J(x_0, u(\cdot)) = \int_0^T L(x(t), u(t))dt + M(x(T)), \text{ where } x(0) = x_0$$

2. consider that the state $x(t)$ of the system, or some parameters of the model, are not know but the variable y can be measured on-line (for instance with sensors), and then construct *observers*, that are systems of the form

$$\frac{d\xi(t)}{dt} = l(\xi(t), u(t), y(t)), \quad \hat{x}(t) = m(\xi(t), y(t)), \quad \xi \in \mathbb{R}^k$$

   so that the estimation error $t \mapsto \hat{x}(t) - x(t)$ converges quickly to 0, whatever are the initial vectors $x(0), \xi(0)$.

This approach is widely used in many application domains including aeronautics, automobile, robotics, chemical reactors, bio-processes.... but surprisingly still very little in epidemiology.

The objective of the internship is to study how available techniques of control theory can be applied for decision making in epidemiology. For instance, consider the SIR model (where S, I and R stand for Susceptible, Infected and Recover populations)

$$\begin{cases} \dfrac{dS(t)}{dt} = -\beta S(t)I(t) \\ \dfrac{dI(t)}{dt} = \beta S(t))I(t) - \rho I(t) \\ \dfrac{dR(t)}{dt}) = \rho I(t) \end{cases}$$

Practitioners typically face two main kinds of problems that can formulated in the framework of control theory :

1. The variable $y(t) = \gamma I(t)$ can represent a ratio of the infected population that is followed in care units, while the parameters β, ρ , γ and the total size of the infected population $I(t)$ is unknown. One aims at reconstructing these unknown quantities as fast possible from the measurements $y(\cdot)$ to better predict and follow the spread of the disease.

2. The infection rate β can be considered as a "control" variable when for instance a social distancing is operated among the population. One then aims to control the size of the infected population by reducing β during a time window.

Many other epidemiological models have been proposed in the literature, most of them being extensions of this simpler SIR model.

This internship is part of the 2024-26 ANR project NOCIME (New Observation and Control Issues Motivated by Epidemiology) : `https://sites.google.com/view/nocime`

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References

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