Internship proposal – Master IDIL

Contribution of Automatic Control to Epidemiology

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In short: Develop "state estimators" and "control laws" for decision making in epidemiology. The considered models will be dynamical systems based on ordinary differential equations, such as SIR, SIS, SIRS... and other models of the literature.

Framework of automatic control

Automatic control consists in considering "input-output" models of the form

$$\frac{dx(t)}{dt} = f(x(t), u(t)), \quad y(t) = h(x(t)), \quad x(t) \in \mathbb{R}^n,\ u(t) \in U \subset \mathbb{R}^p,\ y(t) \in \mathbb{R}^q$$

where $u(\cdot)$ and $y(\cdot)$ are "input" and "output" variables. Two main main goals are

1. design controls law $t \mapsto u(t)$ or feedback controls $x \mapsto \phi(x)$ so that the closed-loop system
$$\frac{dx(t)}{dt} = g(x(t))) = f(x(t), \phi(x(t))$$
possesses *good* properties, such as global asymptotic stability of all the solutions or optimization of a criterion
$$J(x_0, u(\cdot)) = M(x(T)) + \int_0^T L(x(t), u(t))dt, \text{ where } x(0) = x_0$$
2. consider that the state $x(t)$ of the system, or some parameters of the model, are not known but the variable y can be measured on-line (for instance with sensors), and then construct *observers*, that are dynamical systems of the form
$$\frac{d\xi(t)}{dt} = l(\xi(t), u(t), y(t)), \quad \hat{x}(t) = m(\xi(t), y(t)), \quad \xi \in \mathbb{R}^k$$
so that the estimation error $t \mapsto \hat{x}(t) - x(t)$ converges fastly to 0, whatever are the initial vectors $x(0)$, $\xi(0)$.

This approach is widely used in many application domains including aeronautics, automobile, robotics, chemical reactors, bio-processes.... but surprisingly still very little in epidemiology.

Objectives

The objective of the internship is to study how available techniques of control theory [1] can be applied for decision making in epidemiology. Consider for instance the SIR model (where S, I and R stand for Susceptible, Infected and Recover populations)

$$\frac{dS(t)}{dt} = -\beta S(t)I(t), \quad \frac{dI(t)}{dt} = \beta S(t))I(t) - \rho I(t), \quad \frac{dR(t)}{dt}) = \rho I(t)$$

Practitioners typically face two main kinds of problems that can formulated in the framework of control theory:

1. The variable $y(t) = \gamma I(t)$ can represent a ratio of the infected population that is followed in care units, while the parameters β, ρ , γ and the total size of the infected population $I(t)$ is unknown. One aims at reconstructing these unknown quantities as fast possible from the measurements $y(\cdot)$ to better predict and follow the spread of the disease [2].
2. The infection rate β can be considered as a "control" variable when for instance a social distancing is operated among the population [3]. One then aims to control the "peak" of the infected population by reducing β during a time window [4].

Many other epidemiological models have been proposed in the literature, most of them being extensions of this simpler SIR model.

Prerequisites: ordinary differential equations, optimization, numerical integration with Matlab, Scilab, Phyton, Julia...

Some references:

[1] E. Sontag, *Mathematical Control Theory: Deterministic Finite Dimensional Systems*, Springer 2013.

[2] N. Cunniffe, F. Hamelin, A. Iggidr, A. Rapaport, G. Sallet. *Identifiability and Observability in Epidemiological Models - a primer* - Springer Briefs in Mathematics 2024.

[3] A. Rapaport, I. Mimouni. *The role of permanently resident populations in the two-patches SIR model with commuters.* Bulletin of Mathematical Biology, 2023, Vol. 85 (3).

[4] E. Molina, A. Rapaport. *An optimal feedback control that minimizes the epidemic peak in the SIR model under a budget constraint.* Automatica, 2022, Vol. 146.

This internship is part of the research project NOCIME (New Observation and Control Issues Motivated by Epidemiology) funded by the French Research Agency (ANR-23-CE48-0004): https://sites.google.com/view/nocime