Energy transfer from wind to waves : how far does it go ?

Christophe Chaubet + Norbert Kern (+ Miguel Manna)

Laboratoire Charles Coulomb

christophe.chaubet@umontpellier.fr norbert.kern@umontpellier.fr

Ocean Waves

are characterized by their frequency ω, their elevation $\hat\eta$ and their wavenumber k:

$$\eta(x,t)=\hat\eta e^{i(kx-\omega t)} . \quad (1)$$

In the linear approximation, they obey the Euler equation, both in air and in water:

$$\frac{\partial \vec v}{\partial t}=\rho\vec g-\vec\nabla p . \quad (2)$$

and appropriate continuity relations at interfaces.
The dispersion relation is obtained by equating the air pressure P_0 with water pressure at the interface, which gives in absence of wind, and in infinite depth:

$$P_0=P_0-\rho_{air}\, g\eta+k\rho_{air}\,\eta c^2 . \quad (3)$$

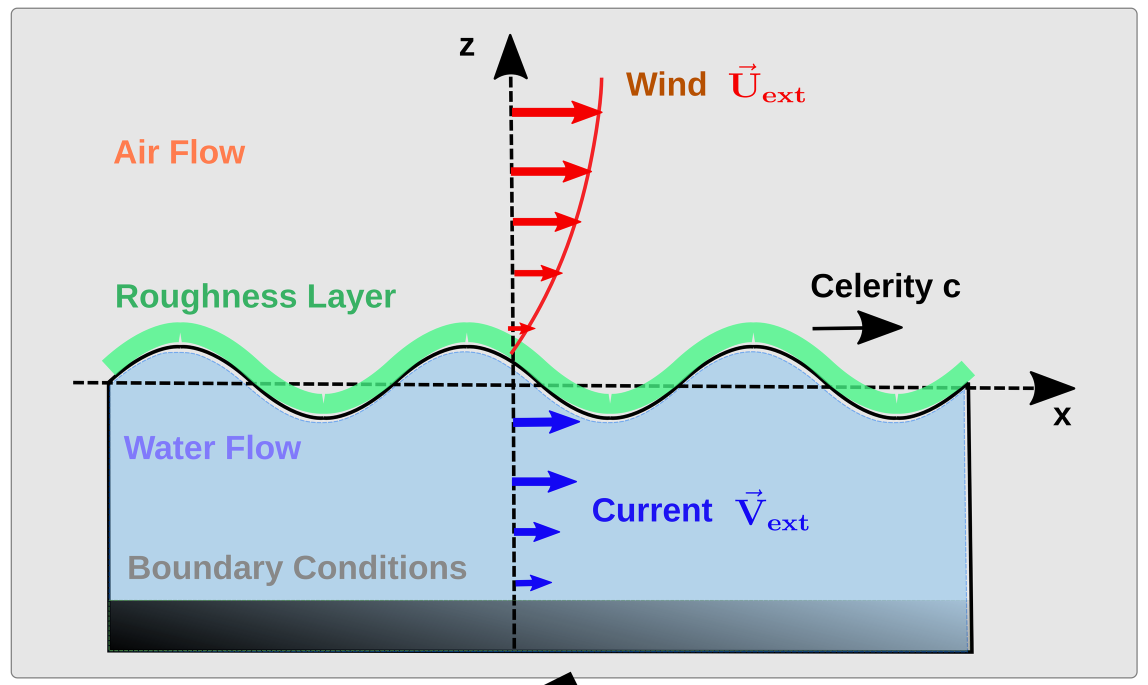
This leads to the following **dispersion relations**:

$$\omega^2=kg\ ;\ c^2=g/k . \quad (4)$$

In the presence of wind, Miles' theory allows to calculate the new air pressure, leading to a complex celerity whose imaginary part yields the **exponential growth** γ :

$$\eta(x,t)=\hat\eta e^{i(kx-\omega t)}e^{\gamma t} , \quad (5)$$

which characterises wind-wave energy transfer.



Other hydrodynamic context

Generalisation to more complex hydrodynamic scenarios are quite recent, accounting for finite water depth [5], shear flow mimicking underwater currents [6], and the role of viscosity [10]. All are of direct interest for **practical applications**, such as the forecast of ocean waves, the growth of waves in the shore regions and the associated issues of coastal erosion, but potentially also for installing sea-based wind farms. Generalising Miles' is involved (analytically and numerically)
Recently we have shown that Miles'approached, initially proposed for a still and infinitely deep ocean of inviscid water, is in fact generic: it can easily be adapted from the mathematical structure of the arguments put forward by Miles [11]. Wave growth rates in complex hydrodynamic situations can be inferred directly from those in Miles conditions. The corresponding conversion factors are determined from the hydrodynamic water pressure produced by a propagating surface wave.
To do this we compare two contributions: the dynamic contribution to the Archimedian water pressure at the interface, on one hand, and its equivalent for Miles hydrodynamics, on the other hand, measured through a coefficient $\mathcal{P}$:

$$\mathcal{P}=\frac{p(z=0)}{p_M(z=0)} . \quad (10)$$

The growth rate can then be easily be calculated, by applying a **hydrodynamic coefficient** $\mathcal{X}_0$ to the growth coefficient $\tilde\gamma_M$ in Miles' conditions:

$$\gamma=\chi_0\times\gamma_M \quad \text{with} \quad \chi_0=\frac{1}{\mathcal{P}_0+\frac{c_0}{2}\frac{\partial\mathcal{P}_0}{\partial c_0}} . \quad (11)$$

Importantly (and conveniently!), these coefficients can be determined without wever accounting for wind in the hydrodynamic calculations (hence the subscript '0').

Wind Wave Interaction: Miles' Approach

Wind constitutes an external flow field $U_{ext}(z)$. From the hydrodynamics in the air domain, the spatial air pressure can be expressed as a function of the vertical position through an integral equation:

$$P(x,z=\eta,t)=P_0-\rho_{air}\, g\hat\eta\, e^{i(kx-\omega t)}+k\rho_{air}\, c^2\,\tilde I\,\hat\eta e^{i(kx-\omega t)} \quad (6)$$

The non-dimensional integral $\tilde I$ is, following Beji and Nadaoka [3]:

$$\tilde I:=\int_{kz_0}^{\infty}\left(\frac{U(z)}{c}-1\right)\frac{u_z(z)}{u_z(z_0)}\,d(kz) \quad (7)$$

where $u_z(z)$ is the air flow perturbation induced by the wave, respecting the boundary condition $u_z(\infty)=0$.
In a first step, Miles shows that the air flow perturbation $u_z(z)$ induced by the wave, obeys the Rayleigh / Orr-Sommerfeld equation [4] in the air domain:

$$u_z''(z)-k^2\,u_z(z)=\frac{U''_{ext}(z)}{U_{ext}(z)-c}\,u_z(z) , \quad (8)$$

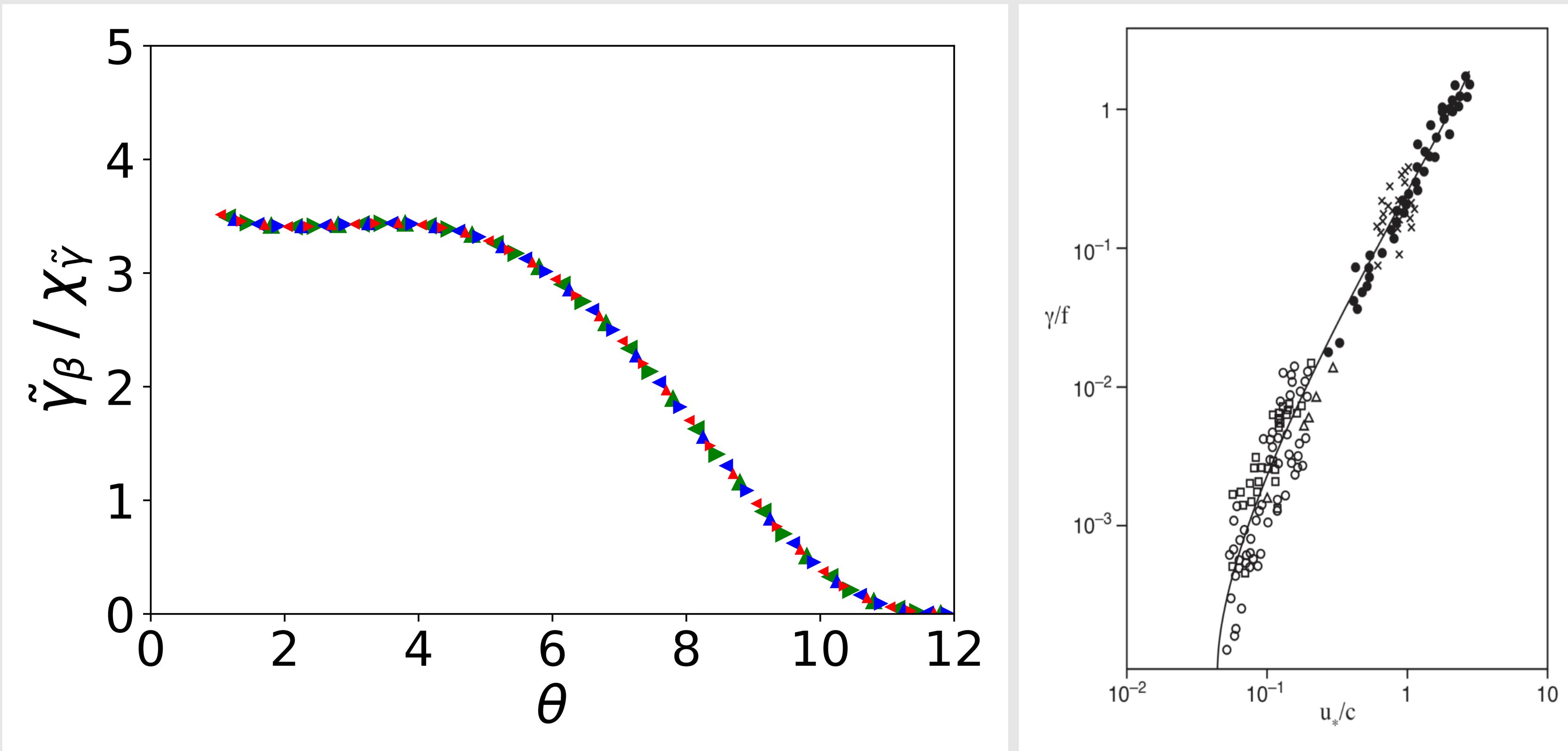
When c is real there is a singularity where the denominator vanishes Beji and Nadaoka [3] have put forward a numerical procedure to solve for the air flow perturbation $u_z(z)$, which yields the integral $\tilde I$, from which the wave growth follows as

$$\tilde\gamma_M=\gamma_M/\tilde\omega_0=\frac{s}{2}\tilde I \quad (9)$$

in the case of Miles' ocean (infinitely deep, no currents, no viscosity).

Miles growth coefficients γ et β

Miles has defined another growth parameter which is directly linked to the pressure field around the wave: $\beta=\theta^2\,\Im\left(\tilde I\right)$, where θ is the **wave age**, defined as $\theta=c/U$. This coefficient is constant for young seas (brief exposure to wind), but it it severly dropss for intermediate seas, and finally vanishes completely at a wage age of 12.8. This is known as the **developed sea**, which ultimately self-regulates wave growth.
Many measurements have been taken along the twentieth century around the world, on seas oceans and lakes (β as a function of theta, and a log-log plot of γ as a function of $1/\theta$.



As shown by the right panel, measurements of the growth rate in varying conditions confirm Miles theory.

Goal: understanding the developed sea

The purpose of this internship is to study the impact of combined hydrodynamic conditions on the maximum age, which ultimately limits the wave growth.

The **starting point** for this internship will be to understand all the physics exposed above, from first principles to the meaning of every established relation. This involves some elements of hydrodynamics. Our publications below can give an idea.

The **main objective** then is to focus on the developed sea, i.e. on the limiting wave age θ_{max}. A certain number of hydrodynamic conditions have been explored (finite water depth, marine shear current, viscosity), but the question of the limiting wave age θ_{max} has not been analysed systematically. This is even more true when several such effects are combined. Using the approach outlined above [11], studying this question is now largely simplified from a technical point of view. However, extracting the important features of such theoretical predictions - and presenting them in a way interesting in real situations - still requires a lot of effort and insight in the physical phenomena.

In addition, there will be options to extend the topic according to the taste of the student taking up the internship (theoretical, conceptual, numerical or with an orientation towards marine physics and its applications).

Pre-requisites: some notions of hydrodynamics; programming (ideally Python); an open mind and an interest in interdisciplinary modelling

References

[1] J. W. Miles, Journal of Fluid Mechanics 3, 185 (1957).
[2] S. Conte and J. Miles, Journal of the Society for Industrial and Applied Mathematics 7, 361 (1959).
[3] S. Beji and K. Nadaoka, Journal of Fluid Mechanics 500, 65 (2004).
[4] S. Beji and K. Nadaoka, Journal of Fluid Mechanics 500, 65 (2004).
[5] P. Montalvo, J. Dorignac, M. Manna, C. Kharif, and H. Branger, Coastal Engineering 77, 49 (2013).
[6] N. Kern, C. Chaubet, R. A. Kraenkel, and M. A. Manna, Coastal Engineering 170, 103976 (2021).
[7] H. Charnock, Quarterly Journal of the Royal Meteorological Society 81, 639-640 (1955).
[8] B. Kinsman, Wind waves - their generation and propagation on the ocean surface (Dover Publications, 2012).
[9] P. Janssen, The interaction of ocean waves and wind (Cambridge University Press, 2004).
[10] C. Chaubet, N. Kern, and M. A. Manna, Physics of Fluids 36, 092109 (2024)
[11] submitted for publication; availabe on https://arxiv.org/abs/2504.09369v1