RHYTHMIC AUDITORY STIMULATION
MULTISCALE MODELING

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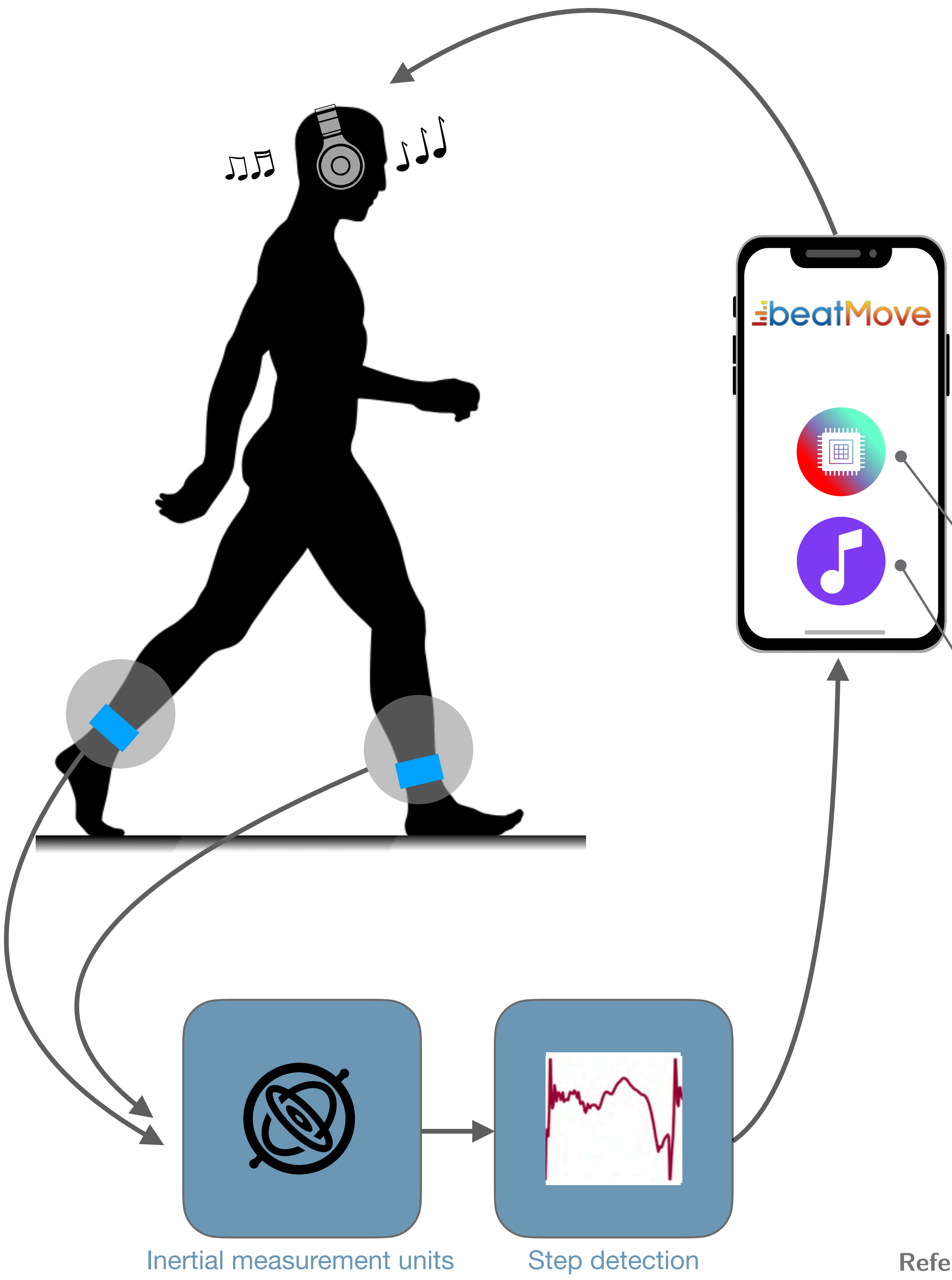
Rhythmic Auditory Stimulation (RAS) is a rehabilitation technique using rhythmic cues to improve motor performance in Parkinson's Disease (PD) [2]. It acts via auditory-motor synchronization to stabilize gait and coordination. RAS has shown benefits in reducing freezing episodes and improving step timing [1].
Through this internship, we will develop a model of this protocol using the framework of coordination dynamics. Namely, we will inspire from the Haken-Kelso-Bunz (HKB) model [3], with a time-dependent forcing signal mimicking the RAS. We will study the dynamic states that such a model is capable of producing, focusing on obtaining a stabilizing effect of the RAS, deriving a map of the parameter space. To this end, we will consider two oscillator models, as described below.



Coupled van der Pol / Rayleigh Model

Inspired by the HKB model [3], we will consider limb coordination via the following coupled oscillator model:

$$\ddot{x}_1 + f(x_1, \dot{x}_1) = (\dot{x}_1 - \dot{x}_2)\,\{A + F(t)\,(x_1 - x_2)^2\}$$
$$\ddot{x}_2 + f(x_2, \dot{x}_2) = (\dot{x}_2 - \dot{x}_1)\,\{A + F(t)\,(x_2 - x_1)^2\}$$

The van der Pol / Rayleigh nonlinearity is given by: $f(x, \dot{x}) = \alpha\,\dot{x} + \beta\,\dot{x}^3 + \gamma\,\dot{x}\,x^2 + \omega^2\,x$. RAS is modeled by modulating coupling: $F(t) = F_0 + F_1\,\sin(\omega t)$.

This captures higher-order coordination effects under external rhythmic forcing that modulates the connection between the two limbs. Indeed, higher-order interactions have recently been proven key in unraveling complex humain brain activity [4]. Hence, we aim to test their effect on sensorimotor dynamics.

Phase Model

We will also work with the standard phase-oscillator version of the HKB model, namely:

$$\dot{\varphi} = -a\,\sin(\varphi) - 2\,b\,\sin(2\,\varphi) + F(t),$$

where $a = \alpha + \beta$, $b = -\beta$ and $F(t) = F_0 + F_1\,\sin(\omega\,t)$. The parameter a and b are in fact coming from the cartesian-coordinate model (1) and expressed as: $a = -(\alpha + \beta\,r^2)$ and $b = \frac{1}{2}\,\beta r^2$. The correspondence between cartesian and phase variables is given by: $x_i = r\cos(\omega\,t + \varphi_i)$ $(i = 1, 2)$, and $\varphi = \varphi_1 - \varphi_2$.

We aim to map parameter domains where RAS promotes movement stability through phase entrainment.

Phase oscillator receiving a coupling term from the gait dynamics in terms of a sine function of the step phase:

$$\frac{d\theta_M}{dt} = \omega + \frac{K}{2}\sin\left(\theta_H(t) - \theta_M(t)\right)$$

ω is the target frequency of the model

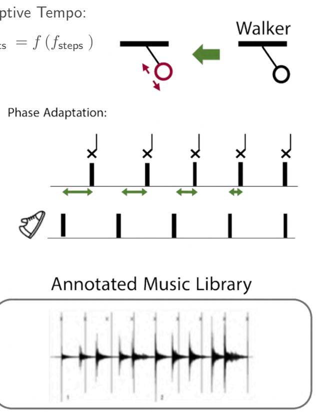
K is the coupling constant,

$\theta_M(\mathrm{t})$ is the phase of the music

$\theta_H(\mathrm{t})$ is the phase of the human gait at time t .

Properties
Adaptive Tempo:

$f_{\text{beats}} = f\,(f_{\text{steps}})$



References

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